大阪大学大学院情報科学研究科情報基礎数学専攻

大学院前期課程入試問題・サンプル

（英語）

【注意事項】

● 問題数は４題である.

解答は各問題ごとに別々の解答用紙に記入すること.
解答用紙は裏面も使用してよい.
解答用紙は未使用や書き損じも含め, すべて提出すること.

● 問題紙は表紙を入れて５枚である. 問題紙は持ち帰ってよい.

（この問題は著作権上問題のない英文を用いた模擬問題である）

［1］次の英文は G. H. Hardy, “A Mathematicians Apology”, Cambridge university press (1940) の一部である．全文を和訳せよ．

Another famous and beautiful theorem is Fermat's 'two square' theorem. The primes may (if we ignore the special prime 2) be arranged in two classes; the primes

$$5, 13, 17, 29, 37, 41, ...$$

which leave remainder 1 when divided by 4, and the primes

$$3, 7, 11, 19, 23, 31, ...$$

which leave remainder 3. All the primes of the first class, and none of the second, can be expressed as the sum of two integral squares : thus

$$\begin{gathered} 5 = 1^2 + 2^2, \quad 13 = 2^2 + 3^2, \\ 17 = 1^2 + 4^2, \quad 29 = 2^2 + 5^2; \end{gathered}$$

but $3, 7, 11$, and 19 are not expressible in this way (as the reader may check by trial). This is Fermat's theorem, which is ranked, very justly, as one of the finest of arithmetic. Unfortunately there is no proof within the comprehension of anybody but a fairly expert mathematician.

［2］次の英文は複素積分について解説したものである．複素平面の上の函数 $f(z)$ に対して，点 a と z を結ぶ曲線が与えられたとき，a から z まで曲線に沿った積分の定義は何か，説明せよ．

The definition of an integral, that is adopted when the variables are complex, is the natural generalisation of that definition for real variables in which it is regarded as the limit of the sum of an infinite number of infinitesimally small terms. It is as follows :–

Let a and z be any two points in the plane; and let them be connected by a curve of specified form, which is to be the path of variation of the independent variable. Let $f(z)$ denote any function of z; if any infinity of $f(z)$ lie in the vicinity of the curve, the line of the curve will be chosen so as not to pass through that infinity. On the curve, let any number of points $z_1, z_2, ..., z_n$ in succession be taken between a and z; then, if the sum

$$(z_1 - a)f(a) + (z_2 - z_1)f(z_1) + \cdots + (z - z_n)f(z_n)$$

have a limit, when n is indefinitely increased so that the infinitely numerous points are in indefinitely close succession along the whole of the curve from a to z, that limit is called the integral of $f(z)$ between a and z. It is denoted, as in the case of real variables, by

$$\int_a^z f(z)\,dz.$$

The limit, as the value of the integral, is associated with a particular curve: in order that the integral may have a definite value, the curve (called the *path of integration*) must, in the first instance, be specified*. The integral of any function whatever may not be assumed to depend in general only upon the limits.

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* This specification is tacitly supplied when the variables are real: the variable point moves along the axis of x.

［注意］vicinity 近傍， tacitly 暗黙に

［出典］A. R. Forsyth, “Theory of Functions of a Complex Variable”, Cambridge university press (1893).

［３］次の英文を読み, 下記の（１）と（２）に答えよ.

Let $\mathbb{R}^2$ denote the two-dimensional euclidean plane. Recall that a *lattice* point is a point $(x, y) \in \mathbb{R}^2$ such that both x and y are integers. A *convex lattice polygon* is a convex polygon any of whose vertices is a lattice point.

Pick's theorem provides a formula for calculating the area of a convex lattice polygon in terms of the number of lattice points located in the interior of the polygon and the number of lattice points placed on the boundary of the polygon. Stated precisely,

Theorem. *Given a convex lattice polygon $P \subset \mathbb{R}^2$, we write $a(P)$ for the number of lattice points located in the interior of P and $b(P)$ for the number of lattice points placed on the boundary of P. Then the area $S(P)$ of P is*

$$S(P) = a(P) + \frac{b(P)}{2} - 1.$$

Exercise. Let P be the convex lattice polygon which consists of all $(x, y) \in \mathbb{R}^2$ satisfying the inequality

$$\frac{|\,x\,|}{5} + \frac{|\,y\,|}{3} \leq 1.$$

Compute the area of P by using Pick's theorem.

［注意］convex polygon 凸多角形, area 面積, interior 内部, boundary 境界

（１）Pick's theorem の内容を, 簡単な具体例を使い, 日本語でわかりやすく説明せよ.

（２）Exercise を解け. 解答は日本語で書いてもよい.

［4］次の英文を読み, 下記の Question の 1 から 4 に答えよ. 解答は日本語で書いてもよい.

The symbol $\mathbb{Z}$ stands for the set of integers. Let $\mathbb{Z}^{d\times d}$ denote the set of $d \times d$ matrices $A = (a_{ij})_{1\le i,j\le d}$ with each $a_{ij} \in \mathbb{Z}$. A matrix $A \in \mathbb{Z}^{d\times d}$ is called *unimodular* if $\det(A) = \pm 1$. A matrix $A \in \mathbb{Z}^{d\times d}$ is said to be in *Hermite normal form* if A is nonsingular and lower triangular with $0 \le a_{ij} < a_{ii}$ for all $1 \le j < i$.

Theorem. *Given a nonsingular matrix $B \in \mathbb{Z}^{d\times d}$, there exist a unique unimodular matrix $U \in \mathbb{Z}^{d\times d}$ and a unique matrix $A \in \mathbb{Z}^{d\times d}$ which is in Hermite normal form such that $A = BU$.*

In the above theorem we say that A is the Hermite normal form of B.

［注意］$\det(A)$ は A の行列式, nonsingular 正則な

Question 1 Find $x \in \mathbb{Z}$ such that $U = \begin{pmatrix} 5 & 3 \\ -3 & x \end{pmatrix} \in \mathbb{Z}^{2\times 2}$ is unimodular.

Question 2 List the matrices $A \in \mathbb{Z}^{2\times 2}$ which are in Hermite normal form with $\det(A) = 2$.

Question 3 Find the Hermite normal form of $B = \begin{pmatrix} -1 & 0 \\ -1 & 2 \end{pmatrix} \in \mathbb{Z}^{2\times 2}$.

Question 4 Let B be the same matrix as in Question 3. Find the unimodular matrix U such that BU is the Hermite normal form of B.